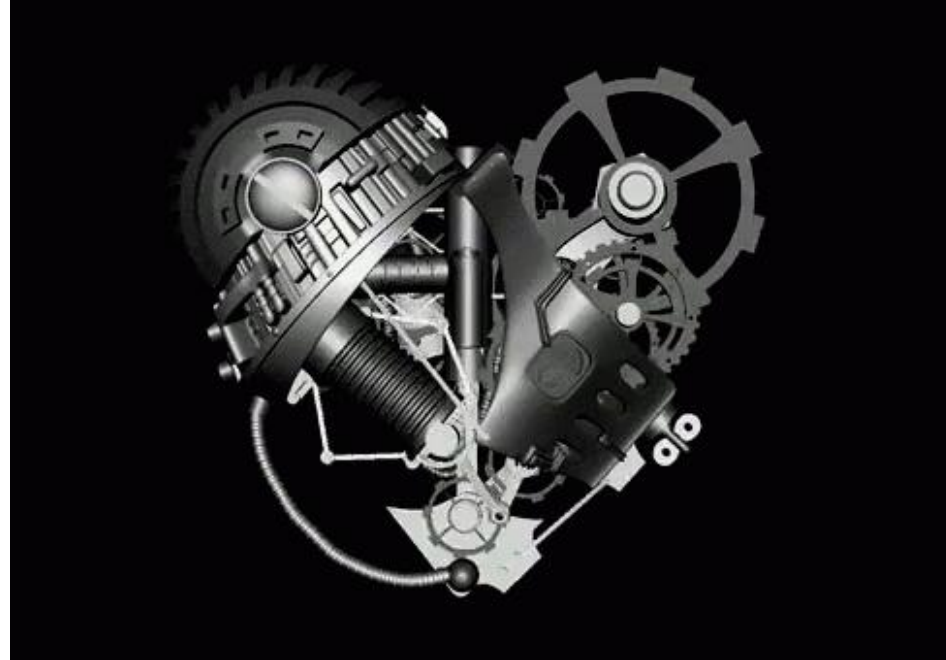


# Grade 12 – Physics



## Unit 1: Mechanics

### Chapter 1: Energy

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## Quiz 1:

## Energy

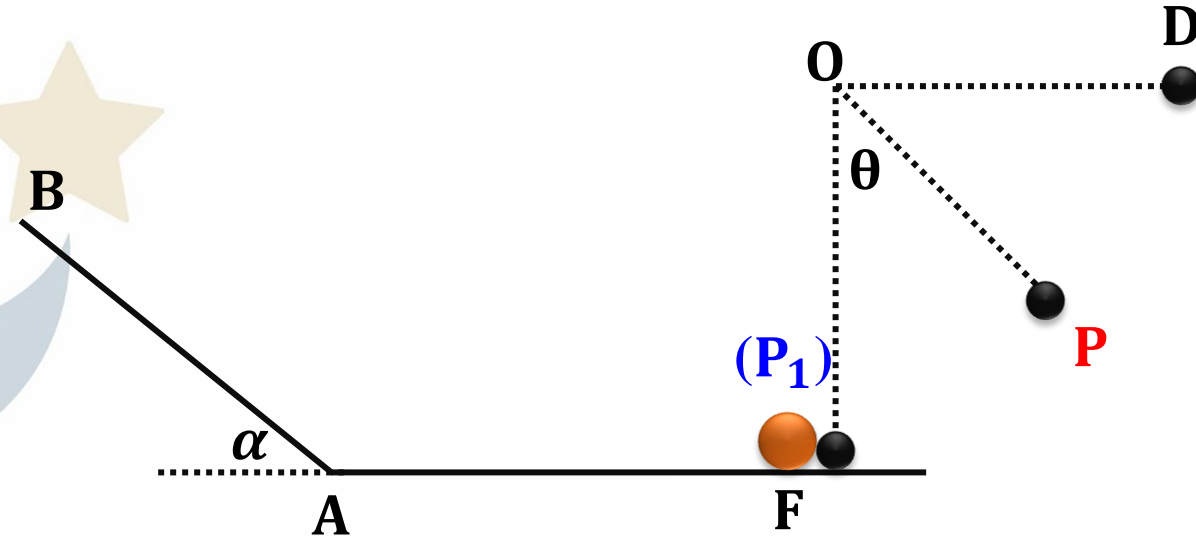
20 min

Consider pendulum is formed of an inextensible and mass less string of length  $l = 0.45m$  having one of its ends fixed while the other end carries a particle (P) of mass  $100g$ .  $g = 10m / s^2$ .

The pendulum is shifted from its equilibrium position by  $\theta_m = 90^\circ$ , then released without initial velocity.

Take the horizontal plane containing FA as a gravitational potential energy reference for the system [(S), Earth].

We neglect friction on the axis through O and air resistance.



1. Calculate the initial mechanical energy of the system [(S),Earth] when (P) was at D.
2. Determine the expression of the mechanical energy of the system [(S),Earth] in terms of  $l, m, g, V$  and  $\theta$ , where  $v$  is the speed of (P) when the string passes through a position making an angle  $\theta$  with the vertical.
3. Determine the value of  $\theta$ , ( $0 < \theta < 90^\circ$ ), for which the kinetic energy of (P) is equal to the gravitational potential energy of the system [(S),Earth].
4. Calculate the magnitude of the velocity  $V_0$  of (P) as it passes through its equilibrium position.

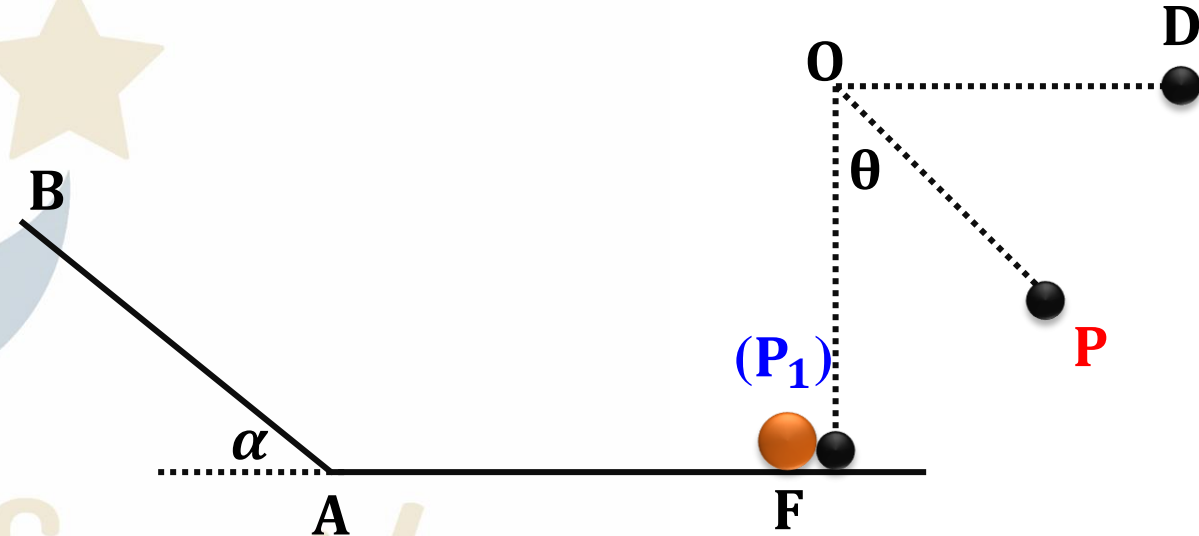
## Quiz 1:

## Energy

20 min

$l = 0.45\text{m}$  ;  $m = 0.1\text{kg}$ ;  $g = 10\text{m/s}^2$ ;  $\theta_m = 90^\circ$ ,  $V_D = 0\text{m/s}$ ;  $f = 0\text{N}$

1. Calculate the initial mechanical energy of the system [(S),Earth] when (P) was at D.



$$ME_D = KE_D + PE_D \Rightarrow ME_D = \frac{1}{2}mV_D^2 + mgh_D$$

$$ME_D = \frac{1}{2}(0.1)(0)^2 + 0.1 \times 10 \times l(1 - \cos\theta)$$

$$ME_D = 0 + 0.1 \times 10 \times 0.45(1 - \cos 90^\circ) \Rightarrow ME_D = 0.45\text{J}$$

## Quiz 1:

## Energy

20 min

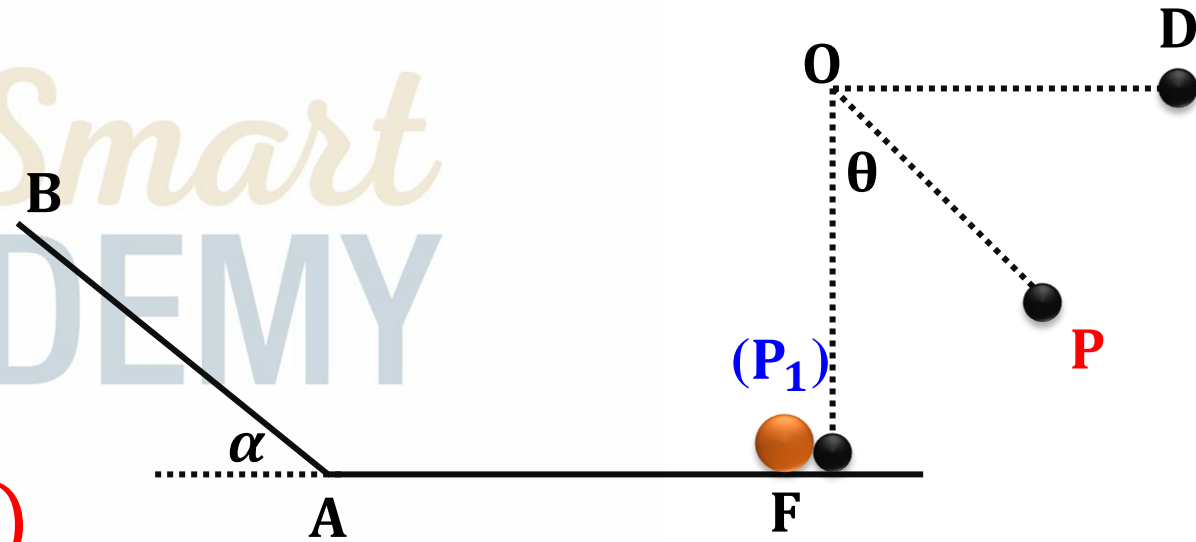
$l = 0.45\text{m}$  ;  $m = 0.1\text{kg}$ ;  $g = 10\text{m/s}^2$ ;  $\theta_m = 90^\circ$ ,  $V_D = 0\text{m/s}$ ;  
 $f = 0\text{N}$

2. Determine the expression of the ME of the system [(S),Earth] in terms of  $l, m, g, V$  and  $\theta$ , where  $v$  is the speed of (P) when the string making an angle  $\theta$  with the vertical.

$$ME = KE + PE$$

$$ME = \frac{1}{2}mV^2 + mgh$$

$$ME = \frac{1}{2}mV^2 + mgl(1 - \cos\theta)$$



## Quiz 1:

## Energy

20 min

$l = 0.45\text{m}$  ;  $m = 0.1\text{kg}$ ;  $g = 10\text{m/s}^2$ ;  $\theta_m = 90^\circ$ ,  $V_D = 0\text{m/s}$ ;  $f = 0\text{N}$

3. Determine the value of  $\theta$ , ( $0 < \theta < 90^\circ$ ), for which the kinetic energy of (P) is equal to the gravitational potential energy of the system [(S),Earth].

The ME is conserved, because friction is neglected; then:

$$ME = ME_D \Rightarrow KE + PE = 0.45\text{J}$$

But given  $KE = PE$  then:

$$PE + PE = 0.45\text{J}$$

$$2PE = 0.45\text{J}$$

$$2mgl(1 - \cos\theta) = 0.45\text{J}$$

$$2 \times 0.1 \times 10 \times 0.45(1 - \cos\theta) = 0.45\text{J}$$

$$\cos\theta = 0.5 \Rightarrow \theta = 60^\circ$$



## Quiz 1:

## Energy

20 min

$l = 0.45\text{m}$  ;  $m = 0.1\text{kg}$ ;  $g = 10\text{m/s}^2$ ;  $\theta_m = 90^\circ$ ,  $V_D = 0\text{m/s}$ ;  $f = 0\text{N}$

4. Calculate the magnitude of the velocity  $V_0$  of (P) as it passes through its equilibrium position

$$ME_0 = KE_0 + GPE_0$$

$$0.45\text{J} = \frac{1}{2}mV_0^2 + mgh_0$$

$$0.45\text{J} = 0.5 \times 0.1V_0^2 + 0.1 \times 10(0)$$

$$0.45\text{J} = 0.05V_0^2$$

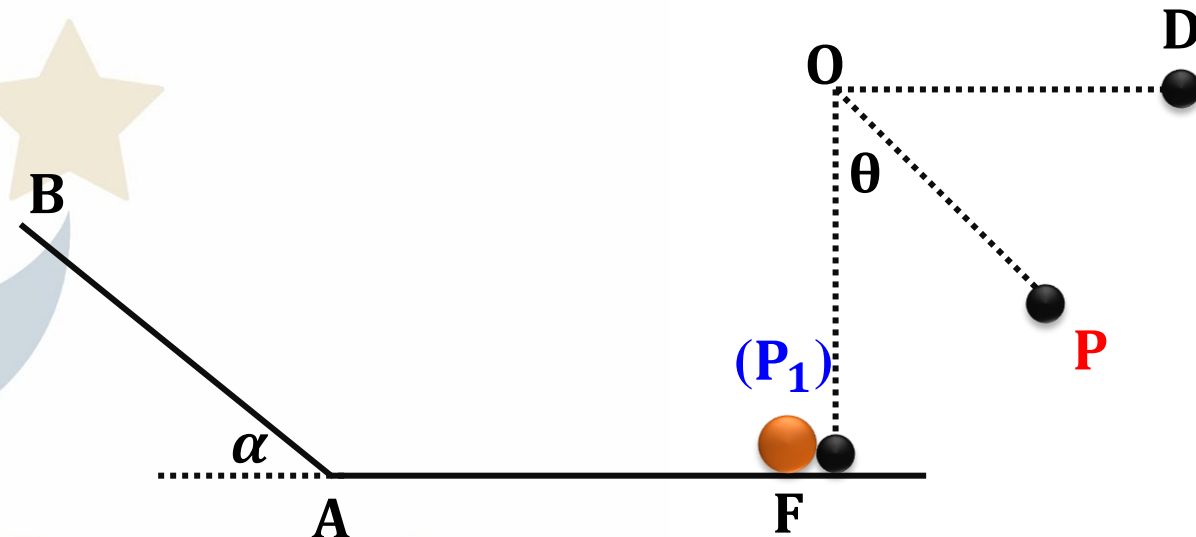
$$V_0^2 = \frac{0.45}{0.05}$$

$$V_0^2 = 9$$

$$V_0 = 3\text{m/s}$$



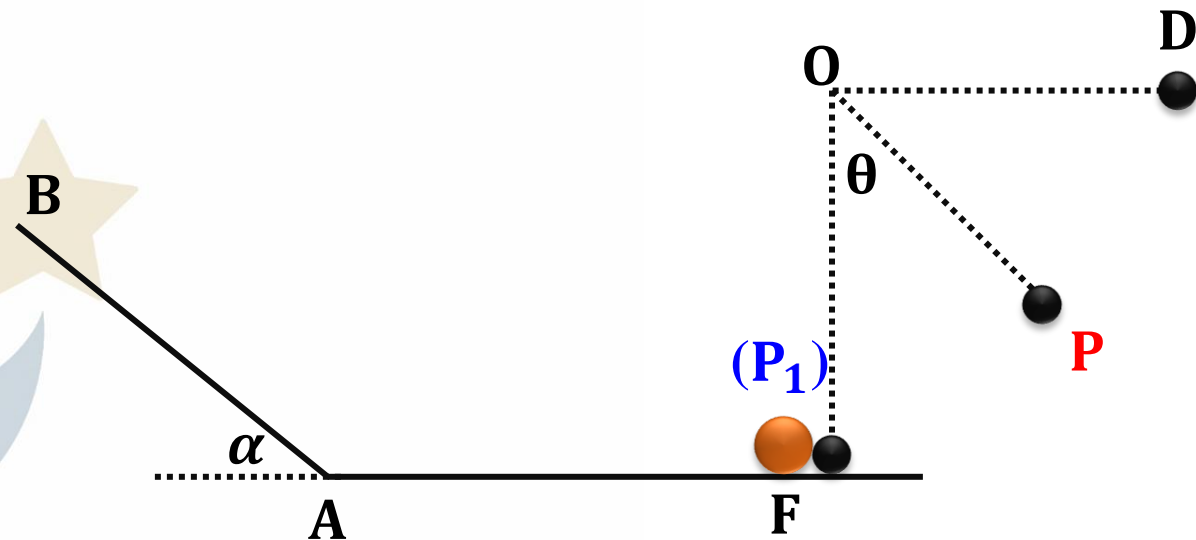
When (P) passes through the equilibrium position, the string is cut, and (P) enters in a collision with a stationary particle ( $P_1$ ) of mass  $m_1 = 200g$ .



As a result of collision ( $P_1$ ) moves along the frictionless horizontal track FA and reaches A with the speed  $V_1 = 2m/s$ . ( $P_1$ ) continues along the line of greatest slope of the inclined frictionless plane AB that makes an angle  $\alpha = 30^\circ$  with the horizontal.

- Determine the altitude of the point M between A and B at which  $(P_1)$  turns back.
- In fact, AB is not frictionless,  $(P_1)$  reaches a point N and turns back, where  $AN = 20$  cm.

Calculate the magnitude of the force of friction (assumed constant) along AN.



## Quiz 1:

## Energy

20 min

$$m_1 = 0.2\text{kg}; g = 10\text{m/s}^2; f = 0\text{N}; V_1 = 2\text{m/s}; \alpha = 30^\circ$$

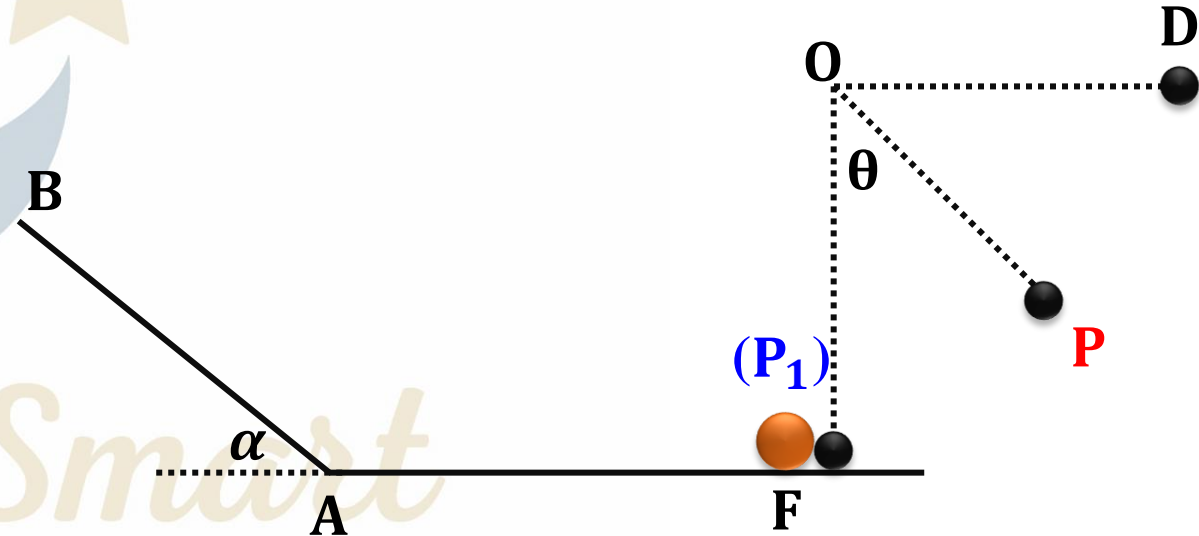
a) Determine the altitude of the point M between A and B at which  $(P_1)$  turns back.

At point M the particle  $(P_1)$  returns then:  $V_M = 0$

$$ME_A = ME_M$$

$$KE_A + PE_A = KE_M + PE_M$$

$$\frac{1}{2}mV_A^2 + mgh_A = \frac{1}{2}mV_M^2 + mgh_M$$



## Quiz 1:

## Energy

20 min

$$\frac{1}{2}m_1V_A^2 + m_1gh_A = \frac{1}{2}m_1V_M^2 + m_1gh_M$$

$$0.5 \times 0.2 \times (2)^2 + 0.2 \times 10 \times (0) = 0.5 \times 0.2(0)^2 + 0.2 \times 10h_M$$

$$0.4 = 2h_M$$

$$h_M = \frac{0.4}{2}$$

$$h_M = 0.2m$$

## Quiz 1:

## Energy

20 min

b) In fact, AB is not frictionless, ( $P_1$ ) reaches a point N and turns back, where  $AN = 20\text{cm}$ .

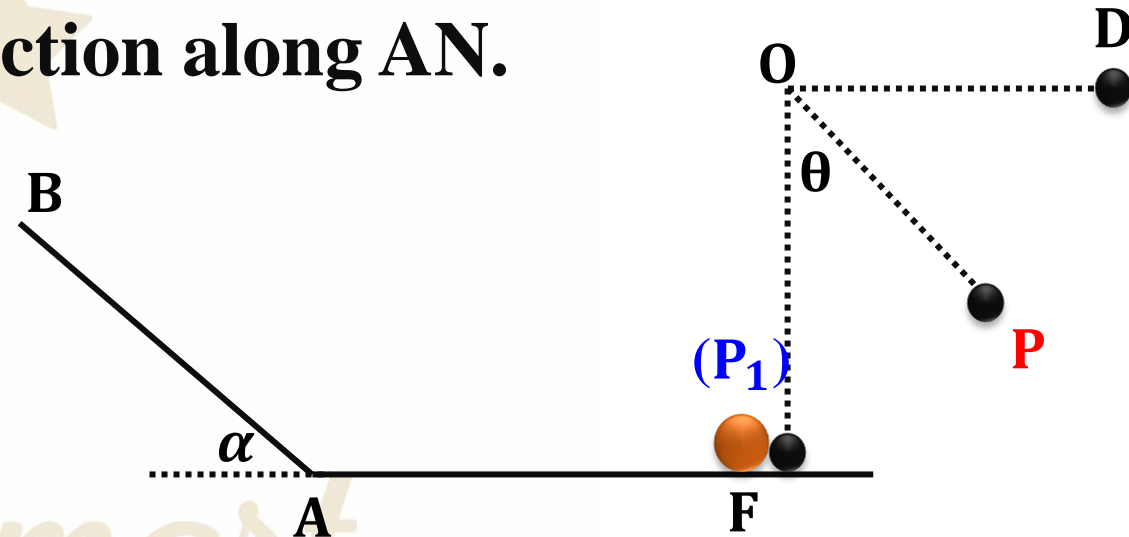
Calculate the magnitude of the force of friction along AN.

$$ME_N = KE_N + PE_N$$

$$ME_N = 0 + mgh_N$$

$$ME_N = 0 + mgAN\sin\alpha$$

$$ME_N = 0.2 \times 10 \times 0.2\sin 30 = 0.2\text{J}$$



$$\sin\alpha = \frac{\text{opp}}{\text{hyp}} = \frac{h_N}{AN}$$

$$h_N = AN\sin\alpha$$

$$\Delta ME_{A \rightarrow N} = W_{\vec{f}_r}$$

$$-0.2J = -f_r \times 0.2$$

$$ME_N - ME_A = -f \times AN$$

$$f_r = 1N$$

$$0.2 - 0.4J = -f \times 0.2$$

Be Smart  
ACADEMY





A particle of mass  $m = 0.2\text{kg}$  is launched at  $t_0 = 0$ , from the ground vertically upward with an initial speed  $v_0$ . Take the ground as a level for gravitational potential energy reference. Use  $g = 10\text{m} / \text{s}^2$

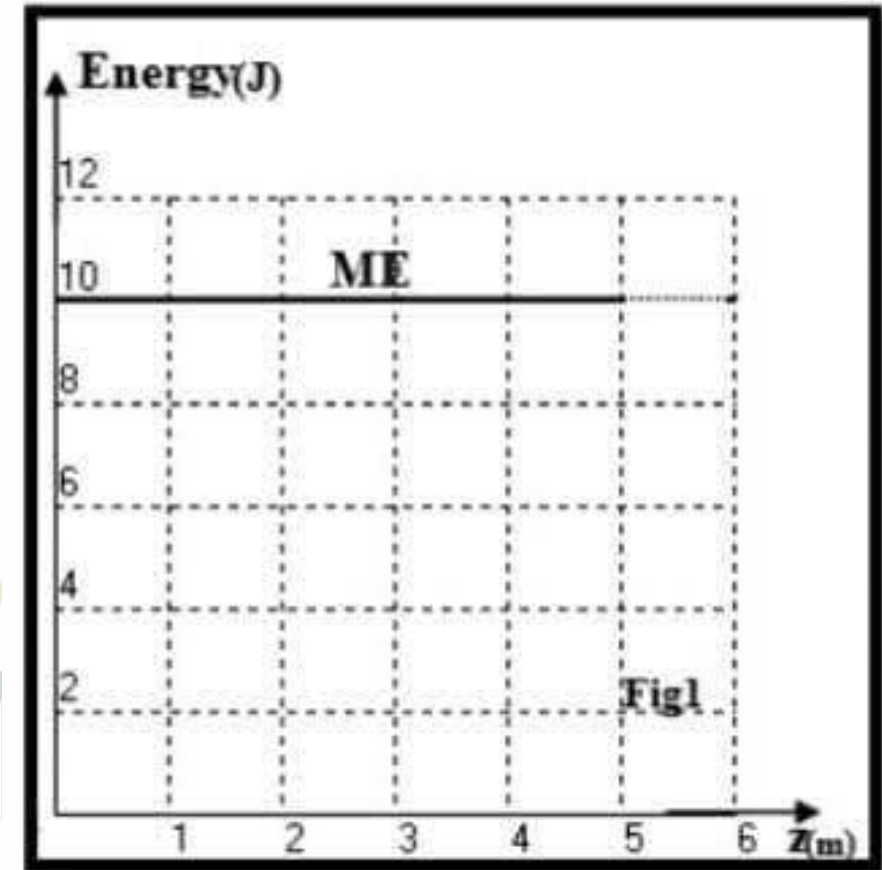
## 1. Motion without air resistance:

Figure 1 shows the variation of the mechanical energy of the system (Particle; Earth;) as a function of the height  $z$  during the ascending phase.

1.1. Use the figure to show that, there are no resistive forces.

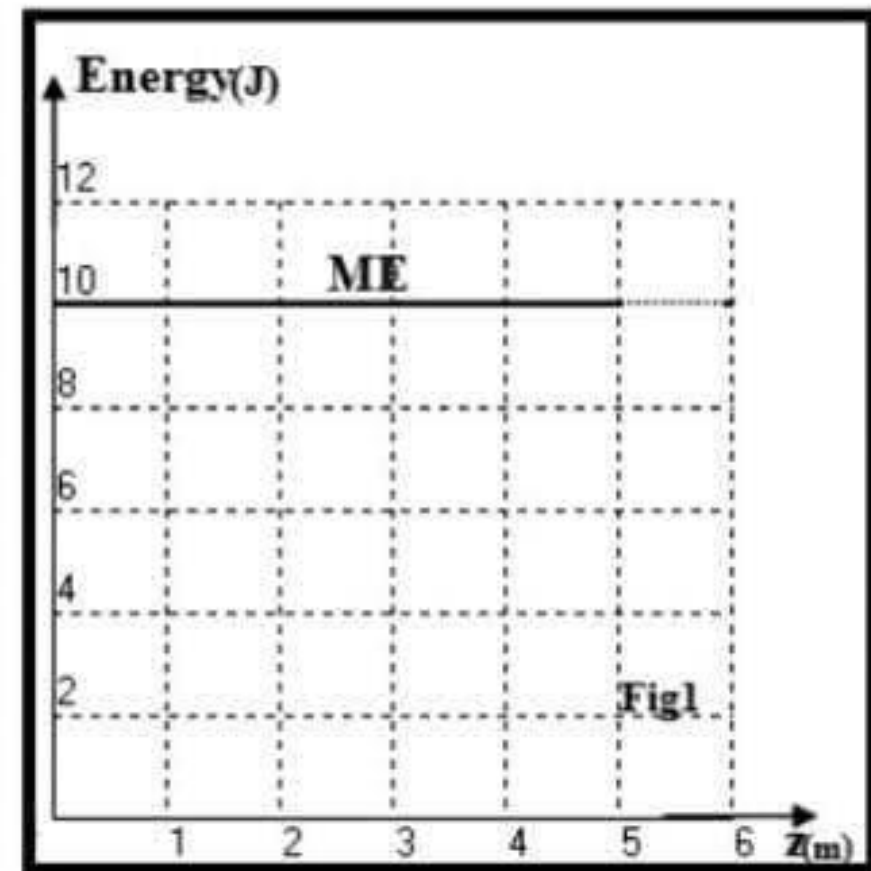
1.2. Determine the initial speed  $v_0$ .

1.3. Write the expression of the GPE as a function of the height  $z$ , and then draw on the same given figure the graph that represents the variation of the GPE as a function of  $z$ .



1.4. Draw on the same given figure the graph that represents the variation of the kinetic energy of the particle.

1.5. Deduce, graphically, the maximum height  $Z_{max}$  reached by the particle.



$$m = 0.2\text{kg}; v_0 = ?; g = 10\text{m} / \text{s}^2$$

## 1. Motion without air resistance:

1.1. Use the figure to show that, there are no resistive forces.

Since the ME is constant at ME=10J; then the ME is conserved.

Therefore, the resistive force is not exist

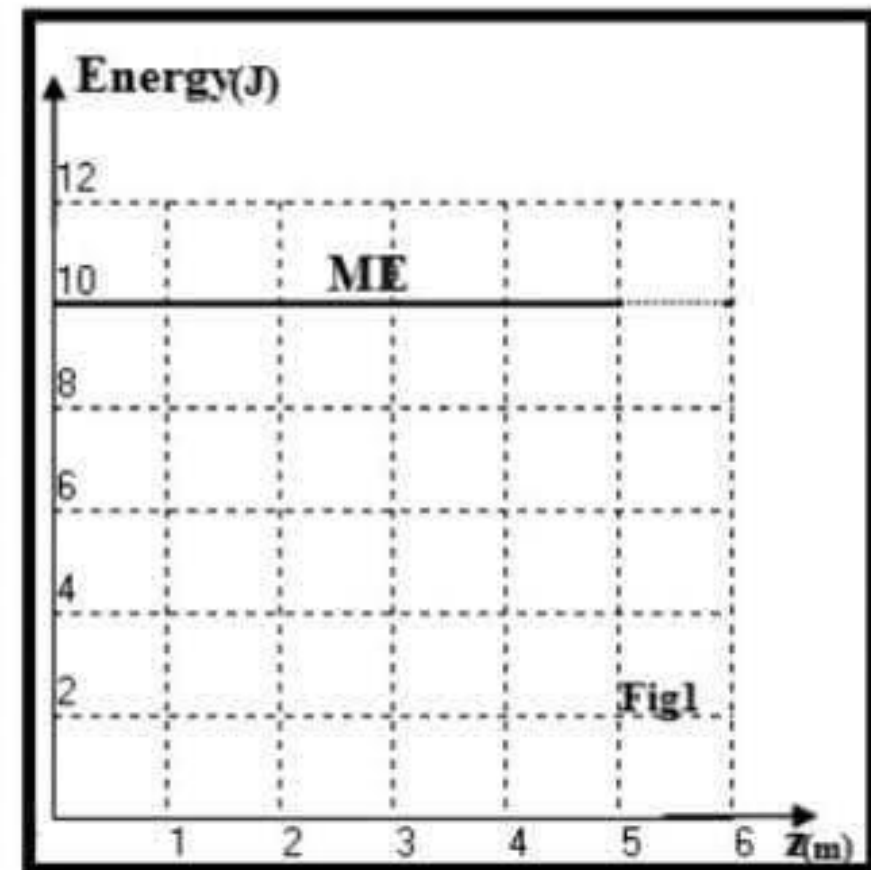
1.2. Determine the initial speed  $v_0$ .

$$\text{ME}_0 = \text{GPE} + \text{KE} \quad \Rightarrow \quad \text{ME}_0 = mgh + \frac{1}{2}mv_0^2$$

$$10 = 0.2 \times 10 \times (0) + \frac{1}{2} \times 0.2v_0^2$$

$$10 = 0.1v_0^2 \quad \Rightarrow \quad v_0^2 = \frac{10}{0.1} = 100$$

$$v_0 = \sqrt{100} = 10\text{m/s}$$



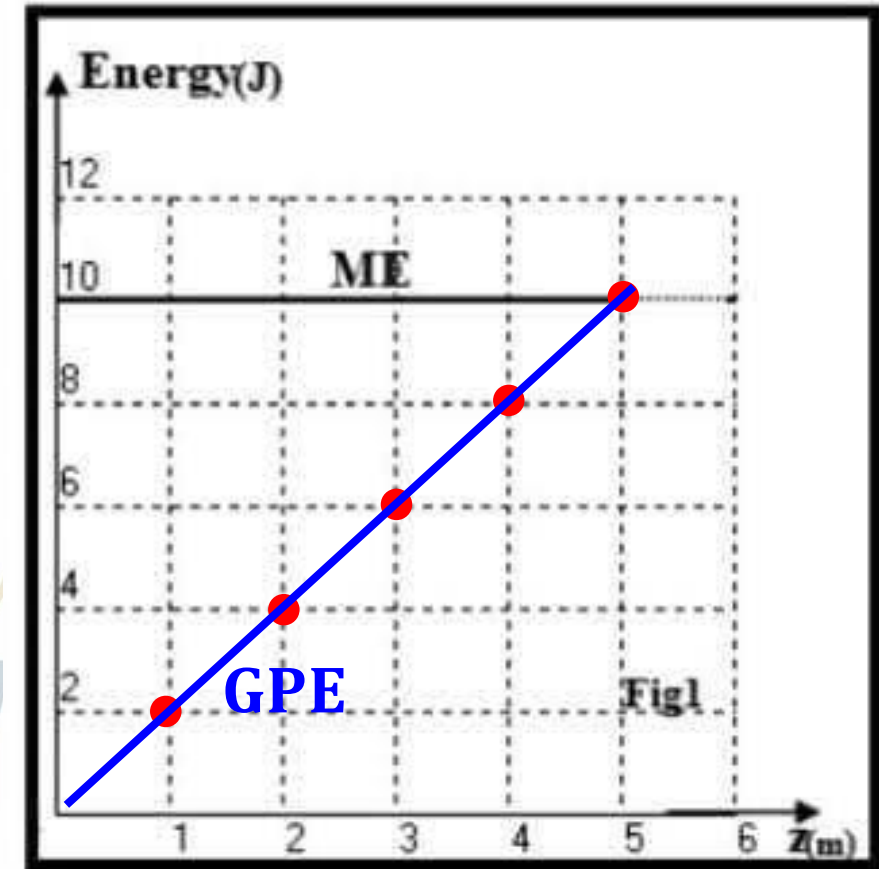
1.3. Write the expression of the GPE as a function of the height  $z$ , and then draw on the same given figure the graph that represents the variation of the GPE as a function of  $z$ .

$$GPE = mgh$$

$$GPE = 0.2 \times 10 \times z$$

$$GPE = 2z$$

$z$	1	2	3	4	5
$GPE = 2z$	2	4	6	8	10

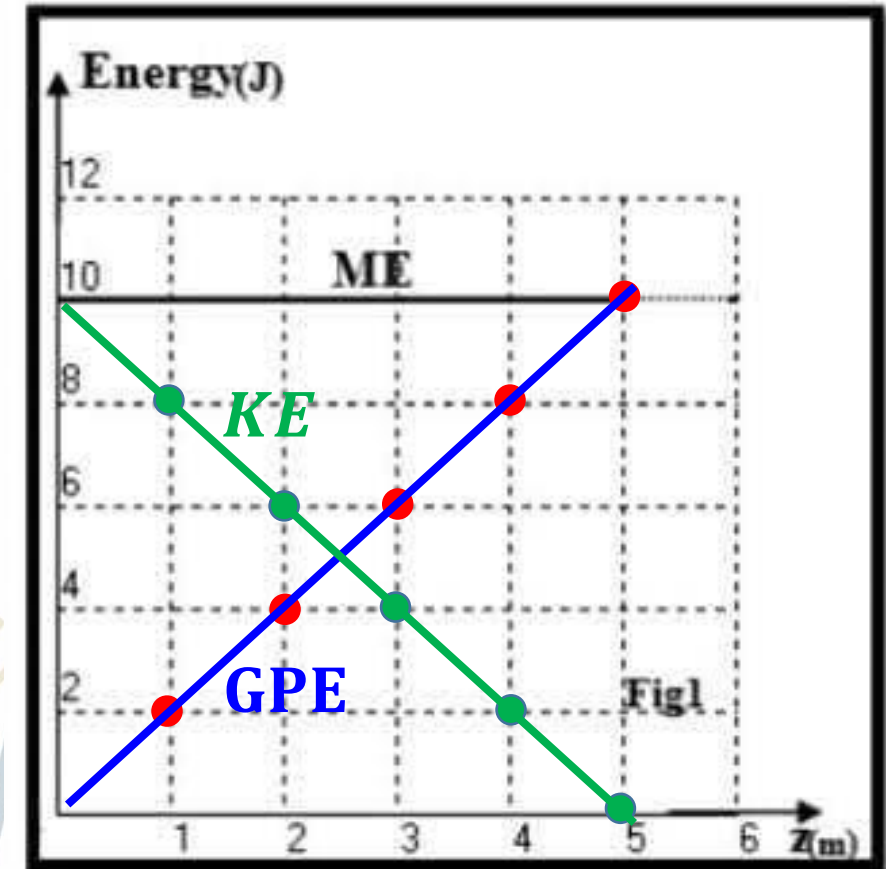


1.4. Draw on the same given figure the graph that represents the variation of the kinetic energy of the particle.

$$ME = GPE + KE$$

$$10 - GPE = KE$$

z	1	2	3	4	5
<b>GPE = 2z</b>	2	4	6	8	10
<b>KE</b>	8	6	4	2	0

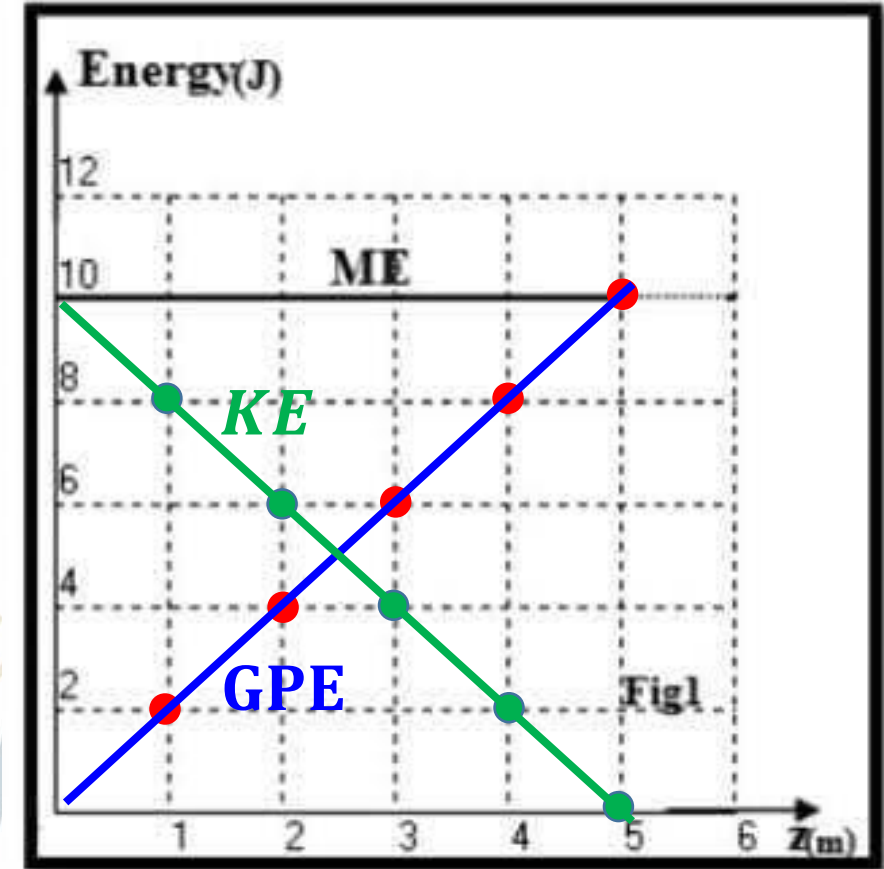


1.5. Deduce, graphically, the maximum height  $Z_{max}$  reached by the particle.

At the maximum height  $Z_{max}$  reached by the particle; the speed become zero ( $v=0$ ).

Therefore,  $KE=0J$ .

From the graph,  $KE=0J$  at  $Z=5m$





**Motion with air resistance:**

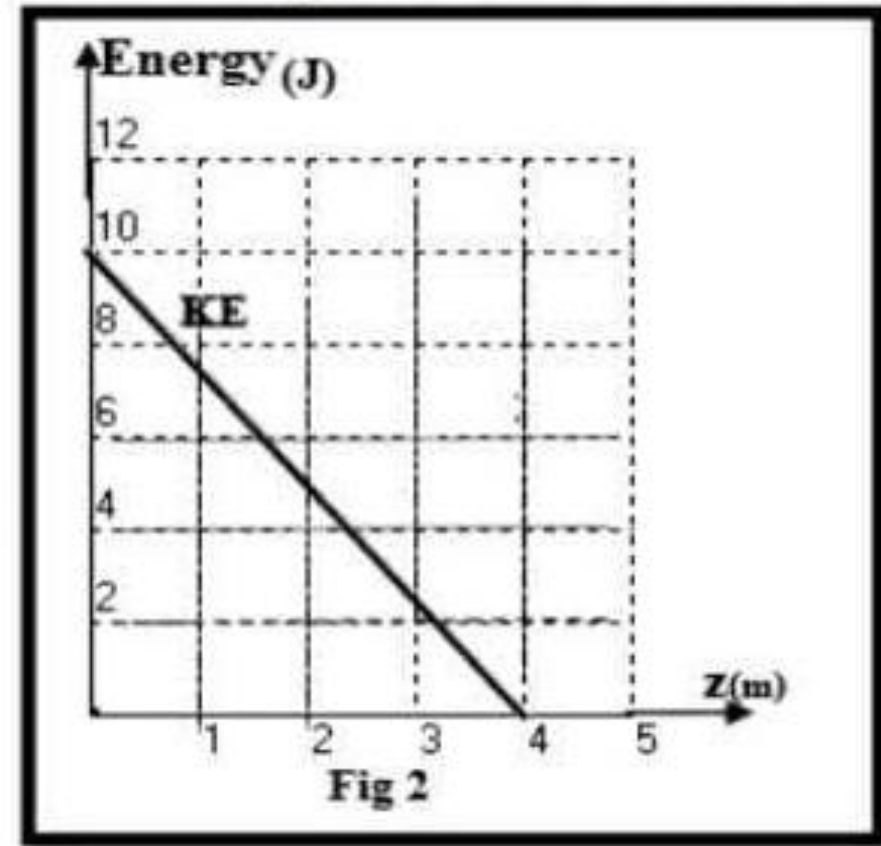
The particle is subjected to air resistance whose direction is opposite to the velocity  $v$  of the particle and whose magnitude is  $f = 0.5N$ .

The adjacent figure (2) represents the variations of the kinetic of the particle as a function of the height  $z$ , during the ascending of the particle.

2.1.Determine the expression of the kinetic energy of the particle as function of  $z$ .

2.2.Show that the expression of the mechanical energy of the system is  $ME = -0.5z + 10$

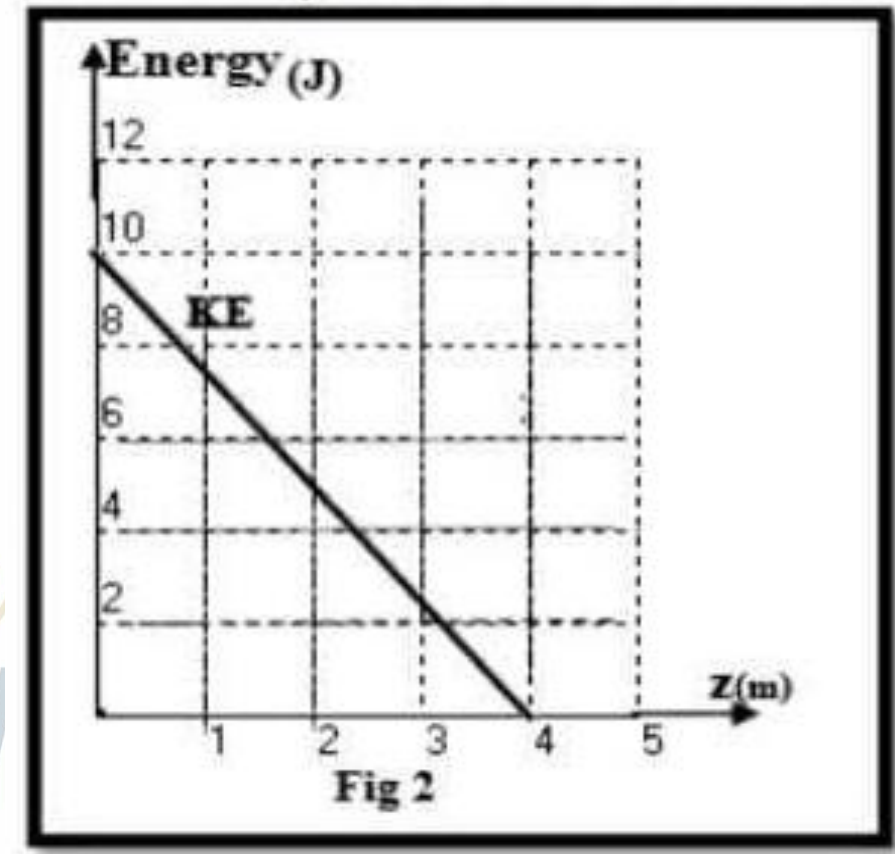
2.3.Determine the maximum height reached by the particle.





2.4. Show that at any instant  $t$  the relation between the algebraic value  $v$  of the velocity of the particle and its altitude  $z$  above the ground is:  $0.1v^2 + 2.5z = 10$

2.5. Deduce the algebraic value of the acceleration of the particle.



2.1. Determine the expression of the kinetic energy of the particle as function of  $z$ .

The graph of KE is S.t line of general equation:

$$KE = az + b$$

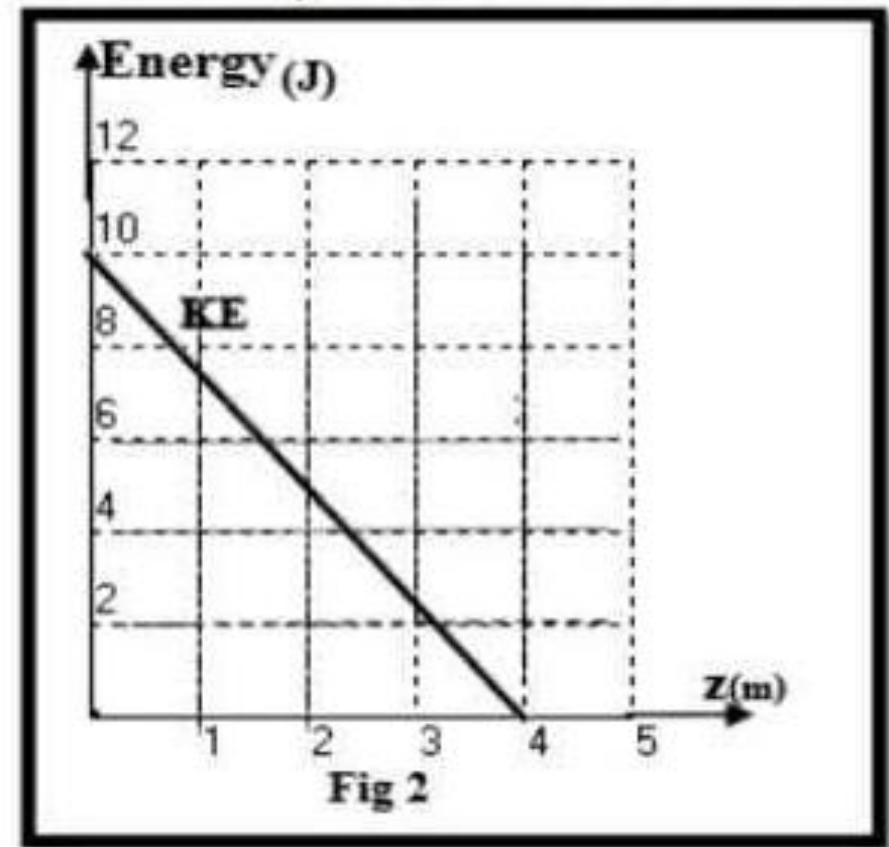
$$a = \text{slope} = \frac{KE_2 - KE_1}{Z_2 - Z_1} = \frac{10 - 0}{0 - 4} = -2.5$$

The constant  $b$  is the y-intercept:

$$b = 10$$

$$KE = az + b$$

$$KE = -2.5z + 10$$



2.2. Show that the expression of the mechanical energy of the system is  $ME = -0.5z + 10$

$$ME = GPE + KE \rightarrow ME = mgh + -2.5z + 10$$

$$ME = 0.2 \times 10z + -2.5z + 10$$

$$ME = 2z + -2.5z + 10$$

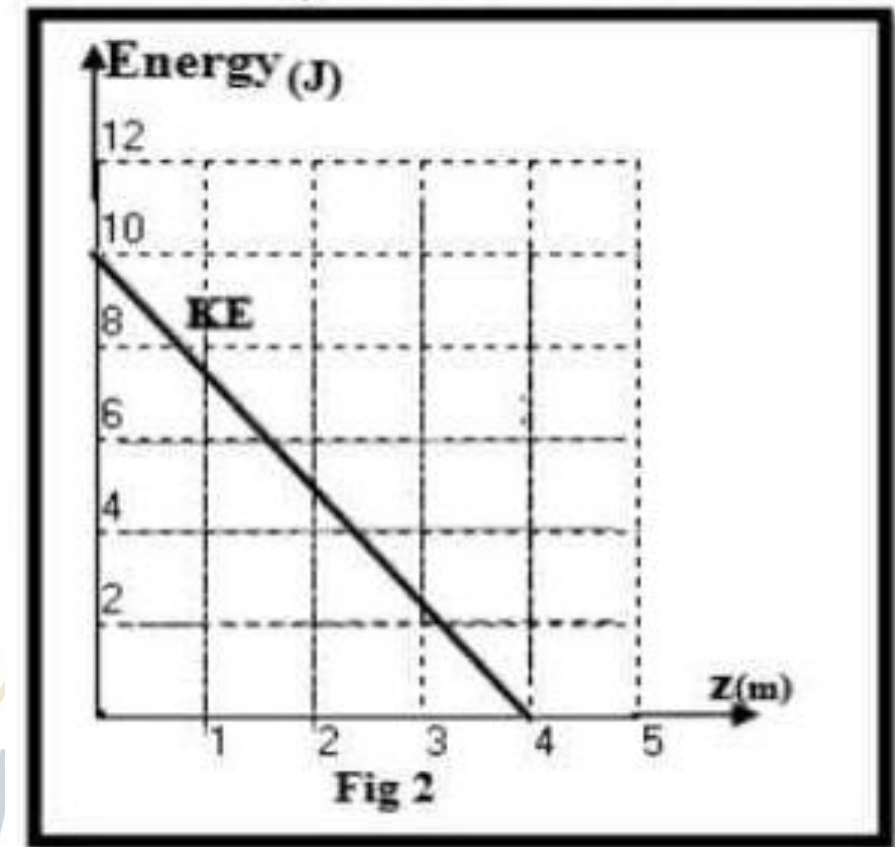
$$ME = -0.5z + 10$$

2.3. Determine the maximum height reached by the particle.

At the maximum height  $Z_{max}$  reached by the particle; the speed become zero ( $v=0$ ).

Therefore,  $KE=0J$ .

From the graph,  $KE=0J$  at  $Z=4m$



2.4. Show that at any instant  $t$  the relation between the algebraic value  $v$  of the velocity of the particle and its altitude  $z$  above the ground is:  $0.1v^2 + 2.5z = 10$

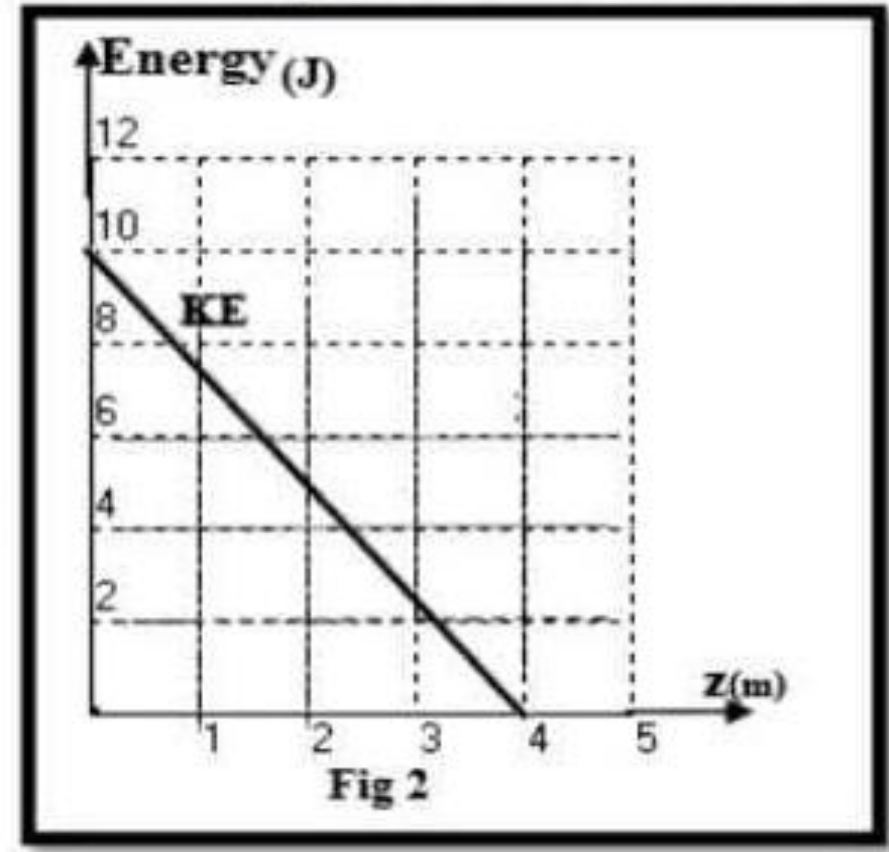
$$\text{ME} = \text{GPE} + \text{KE} \quad \Rightarrow \quad \text{ME} = \frac{1}{2}mv^2 + mgh$$

$$-0.5z + 10 = \frac{1}{2} \times 0.2v^2 + 0.2 \times 10 \times z$$

$$-0.5z + 10 = 0.1v^2 + 2z$$

$$10 = 0.1v^2 + 2z + 0.5z$$

$$10 = 0.1v^2 + 2.5z$$



2.5. Deduce the algebraic value of the acceleration of the particle.

$$10 = 0.1v^2 + 2.5z$$

Derive the above expression w.r.t

$$0 = 0.1[2v.v'] + 2.5.v$$

$$0 = 0.1[2v.v'] + 2.5.v \dots\dots\dots (\div v)$$

$$0 = 0.22.a + 2.5$$

$$0.22.a = -2.5$$

$$a = \frac{-2.5}{0.22} = -11.36m/s^2$$

*A Very Special*  
*"Thank You!"*